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Parametric Instability and Snap-Through of Partially Fluid-Filled Cylindrical Shells

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Abstract

The aim of the present paper is to study the parametric instability and snap-through buckling of an axially pre-loaded, partially fluid-filled cylindrical shell. The Donnell non-linear shallow shell theory is used to study the nonlinear vibrations of the shell. For this, the Galerkin method is used, together with a suitable expansion that takes into account the main nonlinear interactions, to discretize the shell. The resulting nonlinear equations of motion are solved by numerical integration. The fluid is assumed to be non-viscous and incompressible and its inertial effects on the shell surface are obtained by the potential flow theory. A detailed parametric analysis is carried out to demonstrate the influence of the fluid height within the shell on the parametric instability load and on the snap-through buckling load in the main parametric resonance region. Using bifurcations diagrams, the main bifurcation events associated with these stability boundaries are identified. The influence of the different types of bifurcation and fluid height on the safety is also discussed.

Keywords: Cylindrical shell; fluid-shell interaction; parametric instability; snap-through buckling; partially filled shell.

1. INTRODUCTION

Thin-walled cylindrical shells are found in many industrial applications. It is mainly used to support lateral pressure and axial loads. The parametric instability analysis of empty or fluid-filled cylindrical shell has been the subject of several investigations in the past decades. A detailed review of linear and non-linear shell vibrations, including fluid-shell interaction, can be found in a recent book by Amabili

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(2008). In many engineering application the height of the fluid within the shell may vary continuously. The linear analysis of partially fluid-filled tanks has also been conducted by, among others, Lakis and Paidoüssis (1971), Yamaki et al. (1984), Gonçalves and Batista (1987), Gupta (1995), Mazúch et al. (1996), Gonçalves and Ramos (1996), Amabili et al. (1998) and Kim et al. (2004). To the authors' knowledge little is known on the non-linear vibrations of such shells, in particular the influence of a varying liquid height on their bifurcations and integrity.

So, the aim of the present paper is to study the parametric instability and snap through buckling of an axially pre-loaded, partially fluid-filled cylindrical shell. The Donnell non-linear shallow shell theory is used to study the nonlinear vibrations of the shell. For this, the Galerkin method is used, together with a suitable expansion that takes into account the main nonlinear interactions, to discretize the shell. The resulting nonlinear equations of motion are solved by numerical integration. The fluid is assumed to be non-viscous and incompressible and its inertial effects on the shell surface are obtained by the potential flow theory. A detailed parametric analysis is carried out to demonstrate the influence of the fluid height within the shell on the parametric instability load and on the snap-through buckling load in the main parametric resonance region. Using bifurcations diagrams, the main bifurcation events associated with these stability boundaries are identified. The influence of the different types of bifurcation and fluid height on the safety is also discussed.

2. PROBLEM FORMULATION

2.1. Shell equations

Consider a thin-walled cylindrical shell of length L, radius R and thickness h, as shown in Figure 1. The shell is made of a linear elastic material with Young's modulus E, Poisson coefficient ν and mass density ρ. The three displacement components u, v and w are related to the cylindrical co-ordinate system x, θ and z as shown in Figure 1.

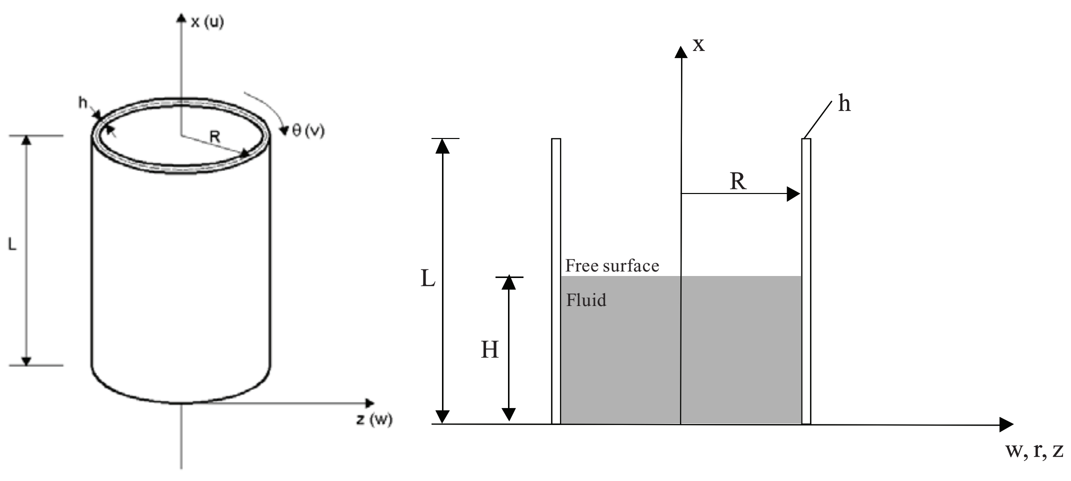


Figure 1: Shell geometry and coordinate system.

These middle surface strain-displacement relations and changes of curvature and torsion are given in terms of the displacement components u, v and w, according to Donnell shallow shell theory, respectively by:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_{x\theta} \end{bmatrix} = \begin{bmatrix} u_{,x} + \frac{1}{2} w_{,x}^2 \\ \frac{1}{R}\left(v_{,\theta} + w\right) + \frac{1}{2R^2} w_{,\theta}^2 \\ v_{,x} + \frac{1}{R}\left(u_{,\theta} + w_{,x}\, w_{,\theta}\right) \end{bmatrix} \qquad \begin{bmatrix} \chi_x \\ \chi_\theta \\ \chi_{x\theta} \end{bmatrix} = \begin{bmatrix} -w_{,xx} \\ -\frac{1}{R^2} w_{,\theta\theta} \\ -\frac{1}{R} w_{,x\theta} \end{bmatrix} \tag{1}$$

The shell is subjected to a harmonic axial load at x = 0, L of the form $P = P_0 + P_1 \cos(\omega t)$ where P_0 is the axial static pre-load (compressive), P_1 is the excitation amplitude, ω is the excitation frequency and t is time.

By using Donnell's nonlinear shallow-shell theory, the equation of motion for finite-amplitude transverse deflection, considering only the transversal inertia and damping forces, are given in terms of the force and moments resultants by:

$$N_{x,x} + \frac{N_{x\theta,\theta}}{R} = 0; \quad N_{x\theta,x} + \frac{N_{\theta,\theta}}{R} = 0 \tag{2a,b}$$

$$\rho h \ddot{w} + \beta_1 \dot{w} + \beta_2 \nabla^4 \dot{w} - p_H - \frac{1}{R^2}\left[R\, M_{x,xx} + \frac{M_{\theta,\theta\theta}}{R} + 2 M_{x\theta,x\theta} \right. \\ \left. R\left(N_x + P_0 + P_1 \cos(\omega t)\right) w_{,xx} + N_\theta \left(\frac{w_{,\theta\theta}}{R} - 1 \right) + 2 N_{x\theta}\, w_{,x\theta} \right] = 0 \tag{2c}$$

where , β_1 and β_2 are, respectively, the linear viscous damping and the viscoelastic material damping coefficients, p_H is the hydrodynamic pressure of the internal fluid on the shell wall and the force and moments resultants are obtained by the integration of the stress components along the shell thickness.

Attention is focused on simply supported, circumferentially closed circular cylindrical shells of length L. For a simply-supported shell, the following boundary conditions must be satisfied:

$$v(0,\theta) = v(L,\theta) = 0 \quad , \quad w(0,\theta) = w(L,\theta) = 0 \quad , \quad M_x(0,\theta) = M_x(L,\theta) = 0 \quad , \\ N_x(0,\theta) = N_x(L,\theta) = 0 \tag{3}$$

The displacement field, in this work, is also required to satisfy the following conditions:

$$u(L/2,\theta) = 0 \text{ and } v(x,0) = v(x,2\pi) \tag{4}$$

In the foregoing, the following non-dimensional parameters have been introduced:

$$W = \frac{w}{h} \qquad \varepsilon = \frac{x}{L} \qquad \tau = \omega_0 t \qquad \Omega = \frac{\omega}{\omega_0} \qquad \Gamma_0 = \frac{P_0}{P_{cr}} \qquad \Gamma_1 = \frac{P_1}{P_{cr}} \qquad P_{cr} = \frac{E h^2}{R\left(3 - 3\nu^2\right)^{1/2}} \tag{5}$$

where P_{cr} is the classical static critical axial load of the shell and ω_0 is the lowest vibration frequency of the empty shell.

2.2. General solution of the shell displacement field by a perturbation procedure

The numerical model is developed by expanding first the transversal displacement *w* trough perturbation techniques in the circumferential and axial variables (Gonçalves and Del Prado, 2005; Gonçalves et al., 2008). The first step in the perturbation procedure is to define a seed modal solution based on physical arguments. The seed mode includes the driven and its companion mode plus a pair of modes containing twice the number of waves in the axial direction as the driven mode to take into account the possible asymmetry of the displacement field in this direction due to the variable liquid height (Amabili, 2008; Gonçalves and Del Prado, 2005; Gonçalves et al. 2008). Based on convergence analyses of the shell response up to large vibration amplitudes, the following modal expansion with eight degrees of freedom, satisfying the boundary conditions, is proposed:

$$\begin{aligned} W = {} & \left[\zeta_{01}(\tau)+\zeta_{11}(\tau)\cos(n\theta)+\xi_{11}(\tau)\sin(n\theta)\right]\sin(q\varepsilon) \\ & +\left[\zeta_{02}(\tau)+\zeta_{12}(\tau)\cos(n\theta)+\xi_{12}(\tau)\sin(n\theta)\right]\sin(2q\varepsilon) \\ & +\psi_{02}(\tau)\left[-\frac{3}{4}+\cos(2q\varepsilon)-\frac{1}{4}\cos(4q\varepsilon)\right] \\ & +\psi_{03}(\tau)\left[-\frac{2}{3}\cos(q\varepsilon)+\cos(3q\varepsilon)-\frac{1}{3}\cos(5q\varepsilon)\right] \end{aligned} \tag{6}$$

where $q=m\pi$ and $\zeta_{ij}(\tau)$, $\xi_{ij}(\tau)$ are the generalized coordinates that are unknown functions of time, *m* is the number of longitudinal half-waves and *n* is the number of circumferential waves. The in-plane displacements *u* and *v* are obtained by substituting equation (6) into the in-plane equilibrium equations, equation (2a, b), and solving the system of partial differential equations in *u* and *v* and imposing the relevant boundary, symmetry and continuity conditions. Finally, by substituting the adopted expansion for the transversal displacement *w* together with the obtained expressions for *u* and *v* into the equation of motion in the transversal direction, equation (2c), and by applying the Galerkin method, a consistent discretized system of ordinary differential equations of motion is derived. An alternative perturbation procedure has been recently proposed by Mallon et al. (2008).

2.3. Fluid equations

The fluid is assumed to be incompressible, and the fluid motion is irrotational so that the flow can be described by a velocity potential, ϕ, which must satisfy the Laplace equation

$$\phi_{,rr}+\frac{1}{r}\phi_{,r}+\frac{1}{r^2}\phi_{,\theta\theta}+\phi_{,xx}=0 \tag{7}$$

The hydrodynamic pressure, p_H, acting on the shell wall can be determined from the linearized Bernoulli equation which can be written as $p_H=-\rho_F\,\dot{\phi}\big|_{r=R}$, where ρ_F is the density of the fluid.
A partially fluid-filled cylindrical shell with a flat rigid bottom is considered, as illustrated in Figure 1, where *H* is the liquid height. At the fluid free surface ($x = H$) the velocity potential is zero, while at the shell bottom ($x = 0$) the axial flow velocity is zero. The motion of shell and fluid is fully coupled by the radial velocities at the fluid-shell interface ($r = R$). So the velocity potential must satisfy the following boundary conditions:

$$\phi = 0\Big|_{x=H}, \quad \frac{\partial \phi}{\partial x} = 0\Bigg|_{x=0} \quad \text{and} \quad \frac{\partial \phi}{\partial r} = \dot{w}\Bigg|_{r=R} \tag{8}$$

By solving equation (14) and imposing the boundary conditions at $x = 0$, H, the following expression is obtained for the potential velocity, as shown by Kim et al. (2004):

$$\phi = \sum_{i=0}^{3} \sum_{\bar{m}=1}^{\bar{M}} \left[A_{i\bar{m}}(\tau)\cos(in\theta) + B_{i\bar{m}}(\tau)\sin(in\theta) \right] \cos\left(\frac{\pi(2\bar{m}-1)x}{2H} \right) \mathrm{I}_{i\times n}\left(\frac{\pi(2\bar{m}-1)rR}{2H} \right) \tag{9}$$

where $\mathrm{I}_{i \times n}$ is the modified Bessel function of first class and order i x n.
The modal amplitudes $A_{i\bar{m}}(\tau)$ and $B_{i\bar{m}}(\tau)$ are determined by applying the impenetrability condition, third condition in equation (8), and by using the Galerkin method and assuming as weighting functions the trigonometric terms in equation (9).
Finally, the hydrodynamic pressure exerted by the contained fluid on the shell is given by:

$$\begin{aligned} p_H = \; -\rho_F \sum_{i=0}^{3} \sum_{\bar{m}=1}^{\bar{M}} \Bigg\{ & \left(\frac{\partial A_{i\bar{m}}(\tau)}{\partial \tau} \cos(in\theta) + \frac{\partial B_{i\bar{m}}(\tau)}{\partial \tau} \sin(in\theta) \right) \times \\ & \cos\left(\frac{\pi(2\bar{m}-1)x}{2H} \right) \mathrm{I}_{i\times n}\left(\frac{(2\bar{m}-1)\pi rR}{2H} \right) \Bigg\} \end{aligned} \tag{10}$$

for $0 < x < H$.

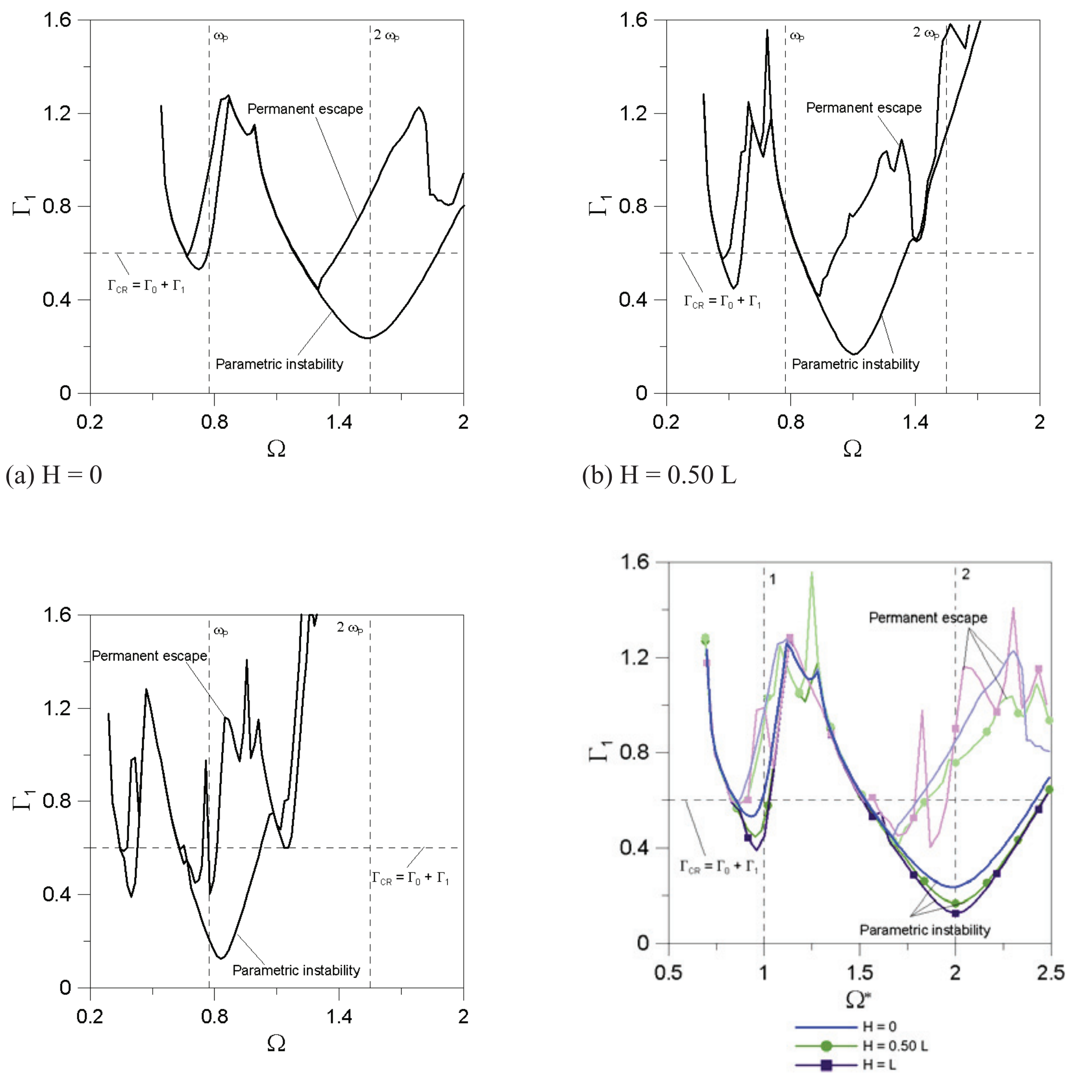
3. Problem formulation

Consider a cylindrical shell of radius $R = 0.2$ m, length $L = 0.4$ m and thickness $h = 0.002$ m. The shell material has the following properties: $E = 210$ GPa, $\nu = 0.3$ and $\rho = 7850$ kg/m³. For this geometry the lowest buckling load and lowest natural frequency are obtained for $m = 1$ and $n = 5$. The linear viscous damping is given by $\beta_1 = 2\eta_1\rho h\omega_0$, with $\eta_1 = 0.0008$, and the viscoelastic material damping is given by $\beta_2 = \eta_2 D$, with $\eta_2 = 0.0001$. The fluid density is $\rho_F = 1000$ kg/m³.

Table 1: Variation of the lowest natural frequencies of the shell with the fluid height (m = 1).

H	$_0$ (rad/s)		
	n = 4	n = 5	n = 6
0.00 L	3745.32	3165.02	3437.18
0.25 L	3533.89	2986.35	3243.12
0.50 L	2676.01	2261.51	2455.96
0.75 L	2138.75	1807.38	1962.78
1.00 L	2004.39	1700.20	1844.27

Table 1 shows the variation of the lowest natural frequencies of the shell with the fluid height, H. The lowest natural frequency always occurs for $m = 1$ and $n = 5$. As shown in previous papers (Gonçalves and Ramos, 1996), the internal fluid reduces significantly the natural frequency of the shell due to the added mass effect.



(a) H = 0
(b) H = 0.50 L
(c) H = L
(d) Superposition of responses

Figure 3: Parametric instability and escape boundaries for the axially pre-loaded cylindrical shell. (Γ_0 = 0.40).

Fig. 3 displays the parametric instability and escape boundaries for a partially fluid-filled cylindrical shell under axial excitation for three different values of the fluid height. The dashed horizontal line shows the critical load of the statically loaded shell, $\Gamma_{CR} = \Gamma_0 + \Gamma_1$. The two vertical lines identify the lowest natural frequency of the shell and twice this value for the empty shell. These correspond to the main resonance regions. The first region around ω_p corresponds to direct resonance, while the second region around $2\omega_p$ corresponds to the main parametric resonance region. The region below the parametric instability boundary corresponds to the stable region of the breathing axi-symmetric mode, which, in the dynamic analysis, is described by a trivial solution of the equations of motion (null perturbation). In the region between the parametric instability boundary and the permanent escape boundary the trivial solution becomes unstable but the amplitudes of the new harmonic solution remains within the pre-buckling potential well. The region above the curve of permanent escape indicates that the response of the cylindrical shell is either within a post-buckling well or exhibits large cross-well motions. The added mass of the fluid leads to a reduction of the natural frequency, so the instability boundaries move to the left. The fluid also decreases the dynamic buckling loads and causes strong variations of the escape boundary due probably to the presence of modal coupling and internal resonances, as shown in Figs. 3a-c. Fig. 3d shows the superposition of the responses shown in Figs. 3a-c. For comparison purposes the forcing frequency in each case, ω, is divided by the natural frequency of the pre-loaded partially filled shell, ω_p ($\Omega^*=\omega/\omega_p$), so that in all cases the resonances occur at $\Omega^*=1$ and $\Omega^*=2$.

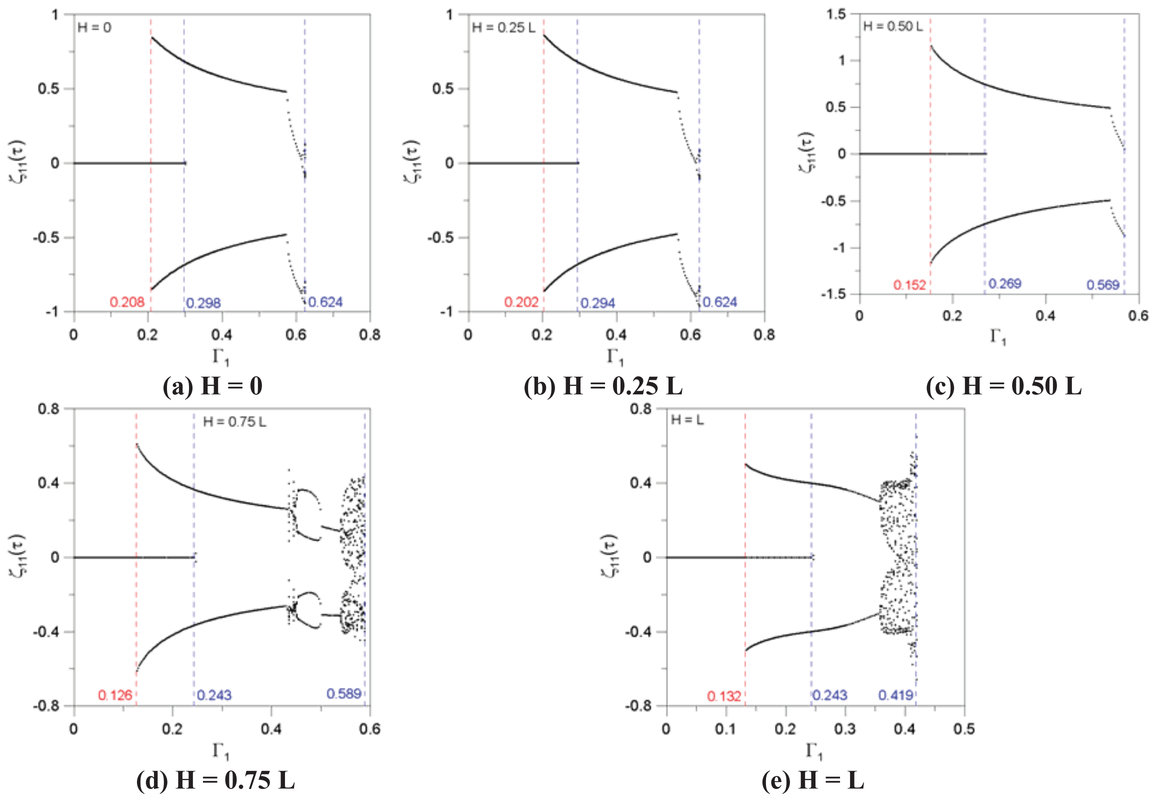


Figure 4: Bifurcation diagrams for a partially filled shell. (Γ_0 = 0.40, Ω^* = 1.825).

Figs. 4 shows the bifurcation diagrams of the cylindrical shell as a function of the forcing amplitude, Γ_1, for selected values of H, with Γ_0 = 0.40. The selected value of the forcing frequency, Ω^* = 1.825, is related to the left of the main parametric instability region. A sub-critical period doubling bifurcation is observed at the critical point followed by several types of bifurcations up to the point where escape (snap-through dynamic buckling) becomes inevitable. However escape may occur for lower Γ_1 values, depending on the initial conditions and shell response transient.

4. CONCLUSIONS

Based on Donnell's shallow shell equations, an accurate low-dimensional model is derived and applied to the study of the nonlinear vibrations of a dynamically loaded fluid-filled circular cylindrical shell in transient and permanent states. The results clarify the influence of the liquid height on the nonlinear dynamic behavior of partially fluid-filled circular cylindrical shells. The ratio between the liquid height and the shell length, H/L, has a marked influence on the parametric and escape boundaries of the shell-fluid system. As the liquid height increases, the stability regions shift to the lower frequency range and the critical loads decrease noticeably in the main resonance regions. Also, a detailed parametric analysis has shown that the liquid height have a marked influence on the bifurcation events and the dynamic buckling load depends on the type of bifurcation and initial conditions. These bifurcations may have a deleterious influence on the stability and integrity of the structure and must be analyzed in detail.

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